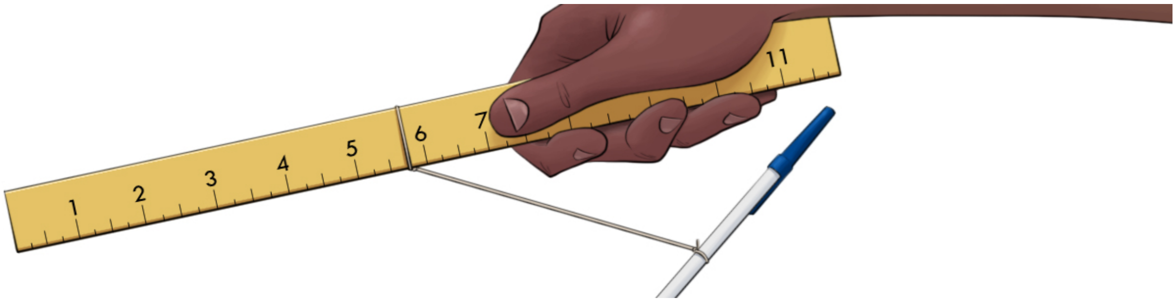


3.3: Total Mechanical Energy

Identify the total mechanical energy of an object as a combination of its gravitational potential energy, elastic potential energy, and kinetic energy.

You have just learned about energy transformations and how energy can be converted from one form to another. Of those different forms of energy, three of them are considered mechanical energy. The sum of an object's gravitational potential energy (GPE), kinetic energy (KE), and elastic potential energy (EPE) is considered mechanical energy. The other forms of energy are nonmechanical in nature. We're going to discuss each of the three types of mechanical energies and learn how to calculate each one.

Kinetic Energy (KE)

The first type of mechanical energy that we are going to learn about is kinetic energy. The word kinetic means motion or movement. Thus, kinetic energy is the energy of motion. Any moving object has kinetic energy. There is a simple formula we can use to determine the amount of kinetic energy an object possesses—kinetic energy equals one-half the mass times the velocity squared. (To get the kinetic energy in joules, it is necessary to have the mass in kilograms and the velocity in meters per second.)

$KE = \frac{1}{2} mv^2$

m = mass measured in kilograms (kg)

v = velocity measured in meters/second (m/s)

Example 3.6:

A 1500 kg car is traveling at 30 m/s. How much kinetic energy does it have?

Show Answer

Gravitational Potential Energy (GPE)

Now that you know how to calculate the amount of kinetic energy that an object possesses, let's take a look at the second type of mechanical energy—gravitational potential energy. *Gravitational potential energy is the energy that an object possesses by virtue of its position within a gravitational field.*

Recall that work is a means of transferring energy to an object. Gravitational potential energy provides an excellent example of how work and energy are related. Gravity is a vertical force. So to change the position of an object within the earth's gravitational field, you must lift or lower the object. This produces a change in height. The force required to raise or lower an object at constant speed must be exactly equal to the force of gravity (F_g = mg). So the work done by this force would equal the force times the distance, or mg times the change in height. This is usually abbreviated as mgh. The units for GPE are the same as the units for work—newton meters or joules.

Here is the formula for gravitational potential energy:

GPE = m · g · h

m = mass measured in grams (g)

g = force of gravity equal to 10 newtons/kg

h = height measured in meters (m)

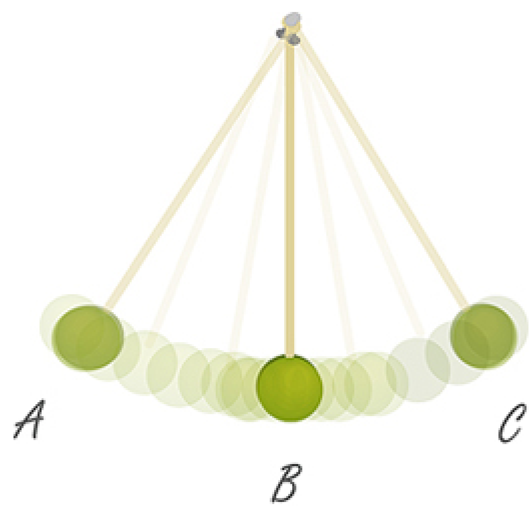


Fig. 3.7: Kinetic and potential energy in a pendulum

It is important to recognize that the h in this equation represents a change in height. For solving problems involving gravitational potential energy, you can simply take the lowest position in a problem to be the "zero height" position. Then when an object is at this position, it would have a height of zero and, therefore, zero gravitational potential energy. For example, a ball at the top of a hill has gravitational potential energy because of its height above the bottom of the hill. If you let it go, that gravitational potential energy will become kinetic energy. Once the ball reaches the bottom of the hill, it cannot go any lower, so that is where we would say the gravitational potential energy is zero.

Note here that a ball at the top of the hill has GPE = mgh (where h is the height of the hill). At the bottom of the hill, the ball cannot go any lower, so the height here is zero and the GPE is also zero. On the other hand, the ball is moving at the bottom of the hill and we know that kinetic energy is the energy of motion. We can therefore conclude that the gravitational energy that the ball possessed at the top of the hill has been converted into kinetic energy.

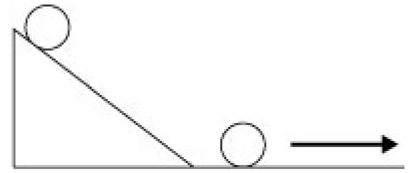


Fig. 3.8: Potential to kinetic energy

Example 3.7:

A 60 kg student is standing at the top of a 20 m tall building. How much gravitational potential energy does the student have?

Show Answer

Elastic Potential Energy (EPE)

Just as energy can be stored gravitationally by virtue of position within a gravitational field, energy can be stored elastically by stretching or compressing elastic substances (such as springs or rubber bands).

Hooke's Law is a principle of physics that states that the that the force needed to extend or compress a spring by some distance is proportional to that distance.

https://phys.org/news/2015-02-law.html

Whenever we discuss springs in this course, we will only be looking at springs that obey Hooke's law ($F_{spr} = kx$). If we look at the work done to stretch a spring a distance of x from equilibrium, we recall that we need to sketch a force vs. displacement graph (where the area of the shape formed by the graph equals the work involved). You can see in the figure below how taking this area leads to an equation for the amount of EPE:

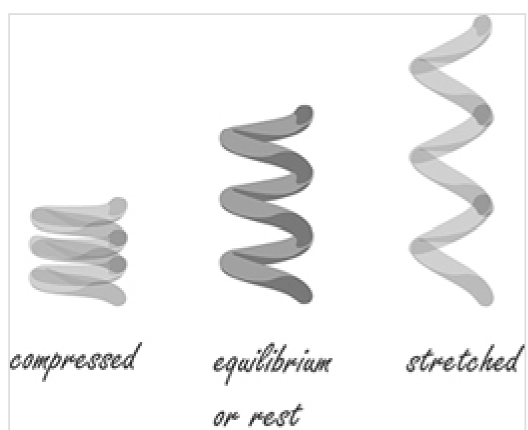
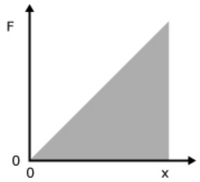


Fig. 3.9: Elastic potential energy in a spring

 Fig. 3.10: Force vs. displacement graph for work done to stretch a spring	• Area = ½ (base) (height) • The *base* = *x* • The *height* = *F*, so • Area = ½ (x) (F) • We can now use substition with Hooke's Law (F = kx), so **Area = ½(x)(kx)**, or, simplified, **½ kx^2** • Thus, **Work = ½kx^2 = EPE** **EPE = ½kx^2**

Therefore the amount of elastic potential energy can be found by taking one-half times the spring constant k times the displacement squared. Here is the formula for elastic potential energy:

EPE = ½ kx^2

k = spring constant measured in newtons per meter (N/m)

x = distance past equilibrium measured in meters (m)

Example 3.8:

A particular spring has a spring constant of 350 N/m. How much elastic potential energy is stored in the spring when it is stretched 1.5 m?

Show Answer

Energy Bar Graphs

We can represent the mechanical energy an object possesses through the use of an energy bar graph. These energy bar graphs can be either qualitative or quantitative. Qualitative graphs have no numbers on them. They show general trends, such as if there is more or less of a certain kind, but not exact amounts of each kind. Quantitative graphs, on the other hand, do show exact amounts and include numbers.

Example 3.9:

Sketch a qualitative energy bar graph for an object that has a little kinetic energy and a lot of gravitational potential energy.

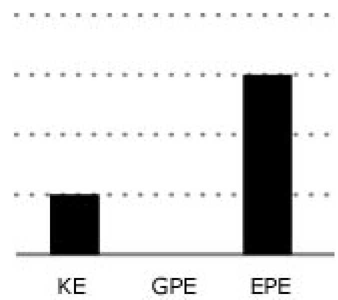
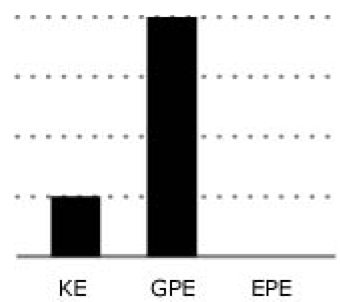


Fig. 3.11: Qualitative graph



Total Mechanical Energy

As was mentioned before, total mechanical energy can be calculated by finding the sum of kinetic energy (KE), gravitational potential energy (GPE), and elastic potential energy (EPE). For example, if we wanted to find the total mechanical energy in in a system where the kinetic energy is 100 joules and the gravitational potential energy is 400 joules, we would add the two together to equal 500 joules of total mechanical energy.