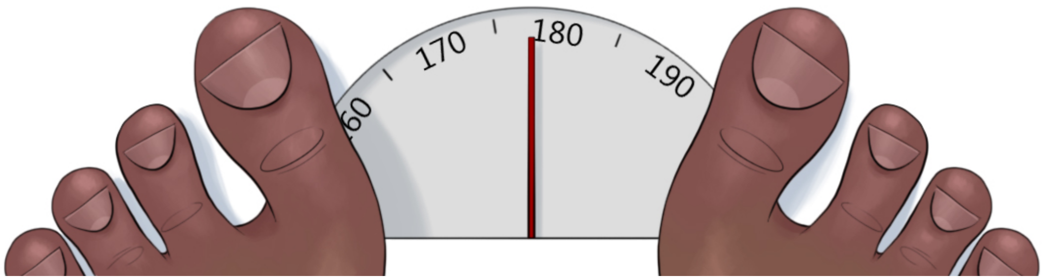


2.5: Newton’s Second Law

Explain the motion of objects using Newton’s second law and solve unbalanced-force problems.

Newton’s first law described what happens when forces acting on an object are balanced. What do you suppose would happen to an object on which unbalanced forces are acting? If you said that the object’s motion would change, you are correct. Recall that Newton concluded that forces cause changes in motion. In lesson 1 we learned about changing motion. We found that changing velocity is called acceleration. So unbalanced forces produce acceleration!

Isaac Newton found a mathematical relationship between the amount of unbalanced force acting on an object and its acceleration. This relationship has come to be known as Newton’s second law of motion. It states that the net force acting on an object is equal to the mass of the object times its acceleration.

Just what do we mean by net force? The word net has many meanings. In the context here, net means remaining. For example, if there is a 100 N force pulling to the right and a 75 N force pulling to the left, then the net force is 25 N to the right. This is how much force is remaining.

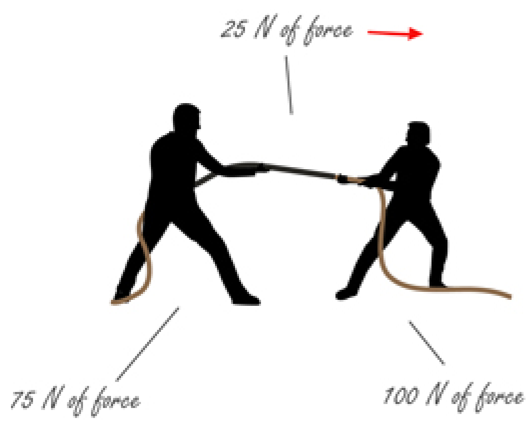


Fig. 2.18: Net force

You can think of net force as the excess, or the amount of force that is left over. In this example, the 75 N pulling to the left could balance 75 of the 100 N of force pulling to the right, leaving 25 N still pulling to the right. There is a mathematical symbol that we use for net force. It looks like this: ΣF. This is a Greek letter sigma in front of the F. It stands for the sum of the forces, or the net force.

To determine the net force on an object we look at the horizontal and vertical directions separately. The net force in the horizontal direction is the difference between the amount of force to the right and the amount of force to the left. Generally, we consider forces to the right to be in the positive direction and forces to the left to be in the negative direction. Then we can find the net horizontal force by taking the forces to the right minus the forces to the left.

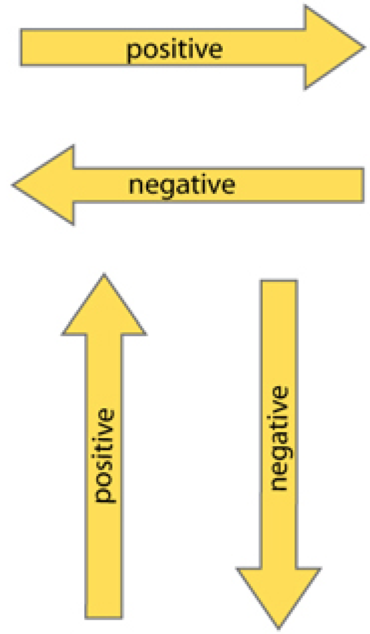


Fig. 2.19: Forces in all directions are considered into the net force.

The net force in the vertical direction is the difference between the amount of force upward and the amount of force downward. Generally, we consider forces upward to be in the positive direction and forces downward to be in the negative direction. Then we can find the net vertical force by taking the forces upward minus the forces downward.

ΣF (Net Force) = positive force - negative force

Fig. 2.20: Formula for net force

In the problems we will see in this course, the net force will be either horizontal or vertical but not both. So for our purposes, when there is an unbalanced (net) horizontal force, we will know the vertical forces are balanced. When there is an unbalanced (net) vertical force, we will know the horizontal forces are balanced. Remember that balanced forces indicate a constant velocity and unbalanced forces indicate acceleration.

Solving Unbalanced-Force Problems

The steps involved in solving unbalanced-force problems are very similar to the steps we used to solve balanced-force problems. Use the following steps to solve example 2.7:

1. Draw a free-body diagram. Label all outside forces acting on the object.
2. Identify the direction the object accelerates (horizontally or vertically). Write out the equations $\Sigma F_x = m \cdot a$ (if acceleration is horizontal) or

$\Sigma F_y = m \cdot a$ (if acceleration is vertical). Leave room to write below.

3. Identify the components of all forces in either the horizontal or vertical direction (whichever is the direction of the net force). Remember that up and right are both positive and down and left are both negative. (We take the positive forces and subtract the amounts of the negative forces; in other words, take up minus down, or right minus left.)
4. Plug in numbers you know, find any missing components, and solve for any unknown values.

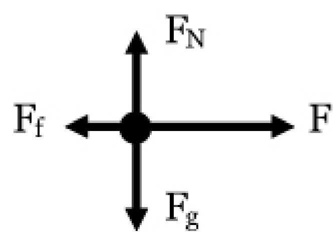
Example 2.7:

A student pushes a 45 kg box across the floor with a 75 N horizontal force. If the box accelerates at 0.6 m/s², how much friction must be present?

Hide Answer

Step 1: Draw the free-form diagram and label it. We call the force the student F because it is a contact force not a special force like tension, friction, or gravity.

Step 2: Because the box accelerates, we know the forces are unbalanced. The box accelerates horizontally, so we use the horizontal forces equation:



$\Sigma F_x = m \cdot a$

Step 3: The force of the student pushing (F) is to the right; friction (F_f) is to the left. The net force (ΣF_x) will be F minus F_f, which will equal mass times acceleration:

$F - F_f = m \cdot a$

Step 4: Plug the numbers we know for F, m, and a into the equation.

$75 - F_f = (45)(0.6)$

Then we solve for friction by adding it to both sides:

$$75 - F_f = 27$$
$$+F_f \quad +F_f$$
$$75 = F_f + 27$$

Then we get F_f on its own by subtracting 27 from both sides:

$48\text{ N} = F_f$

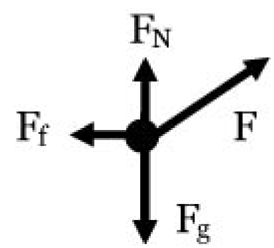
Example 2.8:

Suppose the student pushes the 45 kg box across the floor with the same 75 N force, but this time he pushes at an upward angle of 30°. Again, the box accelerates at 0.6 m/s². Now find the friction force.

Hide Answer

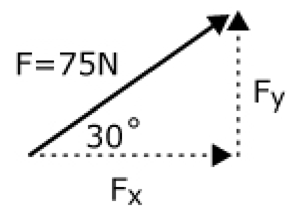
Step 1: Draw the free-form diagram. The student's force (F) is at an angle, so it has x and y components.

Now we need to use what we know about triangles to discover the horizontal force. Draw a triangle and solve for F_x using our sine and cosine formulas (see objective 3 in this lesson for more information).



Fy = 75·sin 30 = 37.5 N

Fx = 75·cos 30 = 65 N



Step 2: Because the box accelerates, we know the forces are unbalanced. The box accelerates horizontally, so we use the horizontal forces equation:

$\Sigma F_x = m \cdot a$

Step 3: The student is pushing to the right (F_x), while friction is pushing to the left. The net force (ΣF_x) will be F_x – friction, which equals mass times acceleration.

$F - F_f = m \cdot a$

Step 4: Plug in the numbers we know for m and a and F_x:

$65 - F_f = (45)(0.6)$

Then solve the equation like we did in the previous example:

$$65 - F_f = 27$$

$$38N = F_f$$

If you found the previous example confusing, keep studying it until it makes sense. You should be able to use this formula to solve for the missing number, even if you are given different information in the question.

Look at example 2.9 to see how to find the missing number, given different information.

Example 2.9:

A 15 kg box is at rest. You push it with 22 N of force at a 30° angle above the horizontal. If there is a 12 N frictional force, what is the acceleration?

Hide Answer

Step 1: Use the same formula construction from example 2.8.

$F - F_f = m \cdot a$

Step 2: Calculate the pushing force (F) using the angle.

(30 cos)(22) = 19.05 N

Step 3: Plug in all the known information and solve for the missing part.

(19.05) - (12) = (15)(a)

7.05 = 15a

$a = 0.47\ m/s^2$