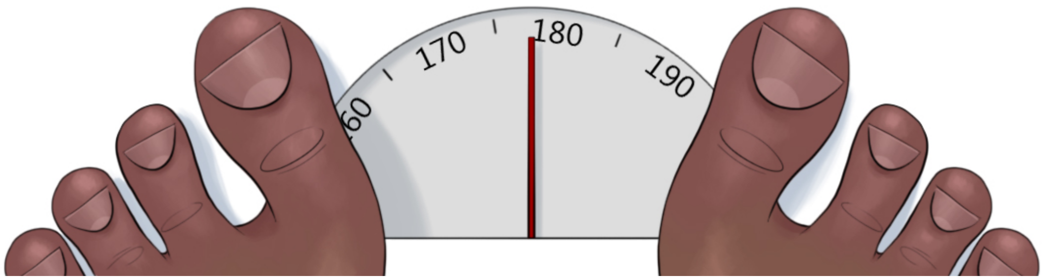


2.3: Force Vectors

Determine horizontal and vertical components of force vectors.

Now that we have learned how to identify the forces acting on an object and represent them with a free-body diagram, we are prepared to analyze situations in which multiple forces are present. Recall from lesson 1 that we learned there was a difference between speed and velocity. What was that difference, again? Oh yeah! Velocity includes direction, while speed does not. There is a name for quantities that include both speed and direction—they are called *vectors.*

Forces are vectors. They have both a size (magnitude) and a direction. For example, I can push a barbell with 75 Kilograms (magnitude) up (direction). I have applied a force to the barbell. Remember how we defined *balanced forces*? We said that forces are balanced if the up forces equal the down forces and the left forces equal the right forces. This works out very well when the forces acting on an object are only in these directions, but it is possible for forces to act in all directions. For example, a force may act at a 45-degree angle to the horizontal. Then it is partly up and partly right.

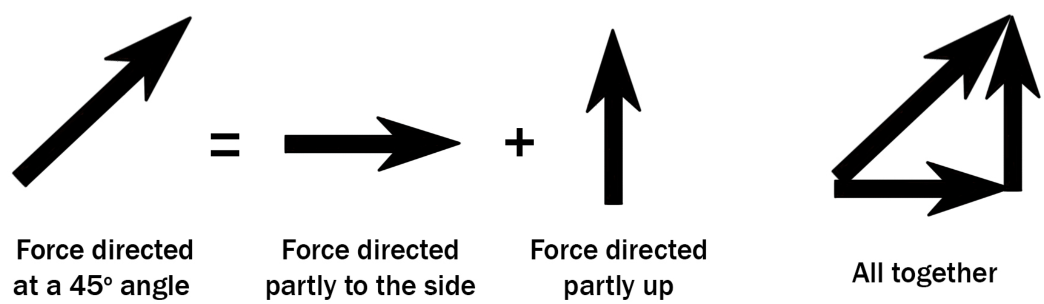


Fig. 2.10: Forces can act in any direction.

Notice how you can break an angled force into horizontal and vertical components. These components will form a triangle. We can determine the amount of up/down and left/right force by understanding a few simple rules about triangles. The three sides of a right triangle have special names. The longest side is called the hypotenuse. The other two sides are called the legs of the right triangle. If one of the angles of the right triangle is known, then the legs have special names too. The leg touching the known angle θ is called the adjacent side. The leg opposite of the known angle θ is called the opposite side. (Generally the Greek symbol theta [θ] is used to represent an angle.)

Now the math. We can determine the length of any two sides of a right triangle by taking the ratio of the opposite and the hypotenuse and the ratio of the adjacent angle and the hypotenuse. These two ratios (ratio is a fancy word for fraction) have special names. They are called the cosine, the sine, and the tangent. See the figure below:

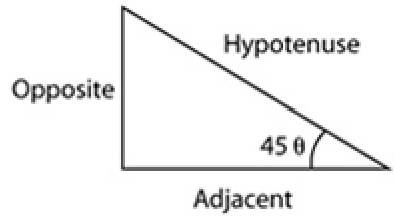
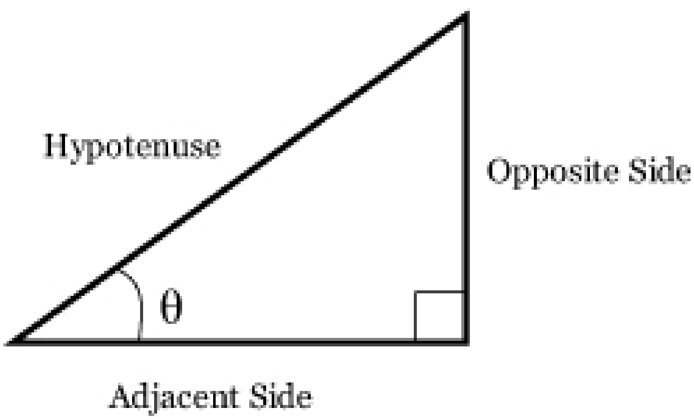


Fig. 2.11: Knowing the different parts of a right triangle is helpful for finding forces at various angles.



$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

Fig. 2.12: Sine, cosine, tangent formulas.

Here's a video that sums this up:

Using a little algebra, we can rearrange these simple equations to solve for the lengths of the sides of the triangle. In this course, we will only be looking at finding the lengths of the legs (the adjacent and opposite sides) of the triangle. You will be given the hypotenuse and the angle. To do this it is only necessary to use the cosine and the sine equations.

To find the adjacent side, take the cosine equation and multiply both sides by the length of the hypotenuse. The hypotenuse on the right side is canceled out, leaving an equation for finding the adjacent side.

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\text{Hypotenuse} \bullet \cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \bullet \text{Hypotenuse}$$

$$\text{Hypotenuse} \bullet \cos\theta = \frac{\text{Adjacent}}{\cancel{\text{Hypotenuse}}} \bullet \cancel{\text{Hypotenuse}}$$

$$\text{Hypotenuse} \bullet \cos\theta = \text{Adjacent}$$

Fig. 2.13: Finding the adjacent side

Similarly, to find the opposite side, take the sine equation and multiply both sides by the length of the hypotenuse. Again, the hypotenuse on the right side gets canceled out and you are left with an equation for finding the opposite side.

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\text{Hypotenuse} \bullet \sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \bullet \text{Hypotenuse}$$

$$\text{Hypotenuse} \bullet \sin\theta = \frac{\text{Opposite}}{\cancel{\text{Hypotenuse}}} \bullet \cancel{\text{Hypotenuse}}$$

$$\text{Hypotenuse} \bullet \sin\theta = \text{Opposite}$$

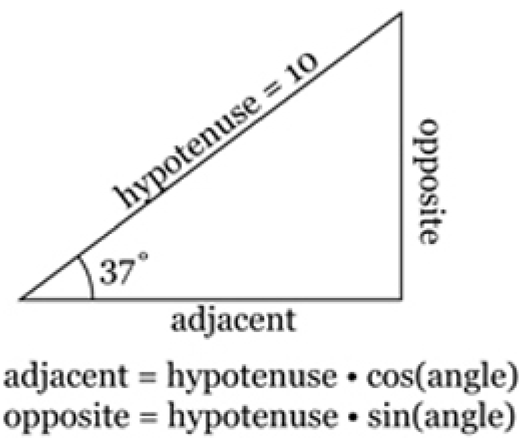
Fig. 2.14: Finding the opposite side

Important side note about calculators: Some calculators do things a little differently. Please check the following so you will know how to use your calculator for this course.

- You need to set up your calculator to be in DEGREE mode. This is usually done by pushing a DRG button so that DEG shows up on the screen, or by pushing a MODE button and making sure the mode is set to DEGREE. Check your calculator manual if you are not sure how to do this.
- Try the following on your calculator: 5 times sin 30 = ____. If you get 2.5 as your answer, then your mode is correct and you know how to find the legs of a right triangle. If not, try this: 5 times 30 sin = ____. Now if you get 2.5, you'll know that this is how you need to enter numbers into the calculator.

Example 2.3:

A right triangle has a hypotenuse of 10 and an angle of 37°. Find the lengths of the adjacent side and the opposite side.



Show Answer

Now that we know how to find the sides of a right triangle, let's apply that knowledge to forces. To find the components of a force, first draw a triangle with the legs of the triangle being horizontal and vertical. This forms a right triangle. We label the legs Fx and Fy. Now just apply our cosine and sine equations to find these legs. Fx = F·cos θ and Fy = F·sin θ.

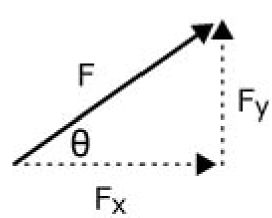
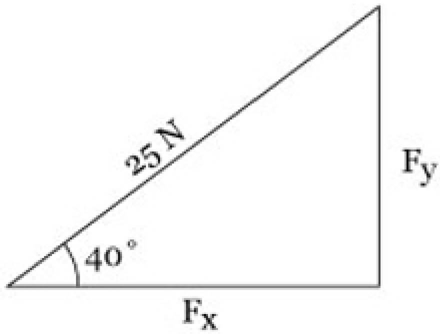


Fig. 2.15: Draw a right triangle to find the components of a force.

Example 2.4:

Suppose a 25 N force acts at a 40° angle. Find the x and y components of this force.



adjacent = hypotenuse • cos (angle)
opposite = hypotenuse • sin (angle)

Show Answer

How do Fy and Fx relate back to the free body diagram you learned about in 2.2? Look at the free body diagram in figure 2.14 for an imaginary object. How do you know if this object is accelerating? You don’t know because you can’t tell if the diagonal force is balanced. However, if you added in the Fy and Fx as I have in figure 2.15, you have the missing forces. Now you can determine if the forces on this imaginary object are balanced.

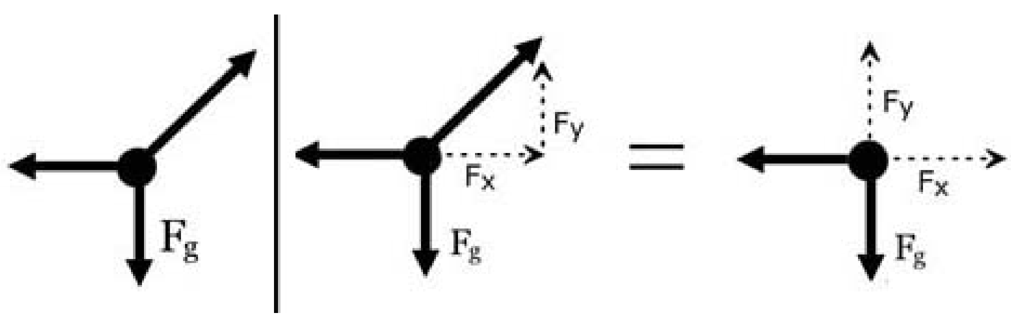


Fig. 2.16.